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Research paper

Teaching mathematical modeling in the classroom: Analyzing the scaffolding methods of teachers

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A R T I C L E I N F O

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A B S T R A C T

Empirical studies demonstrate that students have various difficulties in the mathematical modeling and therefore need teacher support. This research explains teacher support in mathematical modeling through the three characteristics of scaffolding (i.e., contingency, fading, and transfer of responsibility). Studies using this theoretical perspective involve open questions concerning which teacher support is suitable for effective scaffolding methods. This study aims to examine the scaffolding methods of teachers who used different participant structures during mathematical modeling activities. This paper argues that teachers' participation structures relate to their scaffolding methods. It also addresses issues that teachers may reflect on while supporting modeling activities.

1. Introduction

Mathematical modeling is an essential skill in the secondary school mathematics curricula of many countries (Greefrath & Vorhölter, 2016; Stender & Kaiser, 2015). Studies confirm that the mathematical modeling process is complex (Kaiser & Stender, 2013; Stender et al., 2017) and involves various difficulties (Galbraith & Stillman, 2006; Maaβ, 2005). Students hence require the teachers' observation and support to become independent modelers (Stender, 2018; Stender & Kaiser, 2015). No single method to support the mathematical modeling process for teachers exists, but specific criteria are relevant or indispensable (Durandt et al., 2022). Particularly, teachers must have a theoretical background to support students engaged in modeling activities (Borromeo Ferri, 2018). Modeling studies indicate that scaffolding has a strong theoretical base for support (Stender et al., 2017; Stender & Kaiser, 2015). There is an intensive discussion on which the teacher support is suitable for effective scaffolding (Stender & Kaiser, 2015; Tropper et al., 2015). In previous studies (Stender & Kaiser, 2015; Tropper et al., 2015), teachers' scaffolding methods were associated with their experience and training. However, which scaffolding methods foster modeling competencies has not been conceptualized (Bakker et al., 2015). When complex modeling activities are conducted, the teacher's role becomes highly demanding (Stender et al., 2017). Tabak and Baumgartner (2004) stated that the roles of teachers in the classroom relate to participant structures. This study aims to reveal how scaffolding methods in modeling relate to teachers' participant structures. Classifying the scaffolding methods according to different participant structures can be important in conceptualizing scaffolding.

For this study, in-depth interviews on how teachers supported the mathematical modeling process, how their support changed over time, and whether the responsibility was transferred to the students were held with those who used different participation structures. This study seeks to understand, uncover, and detail the scaffolding experiences of teachers who used different participant structures during mathematical modeling activities. Moreover, comparing the scaffolding experiences of teachers who used different participation structures can present an initial idea about the effectiveness of their methods. This study's results can therefore help teachers further refine their methods in teaching mathematical modeling.

This study aimed to argue the scaffolding methods of secondary school mathematics teachers who used different participant structures during mathematical modeling activities, based on the teachers' statements. The research questions were as follows:

1) Which participant structures do teachers use during mathematical modeling activities?
2) What forms of support do they use during mathematical modeling activities?
3) What are teachers' opinions on potentially reducing their support over time throughout modeling activities?
4) What are teachers' perspectives regarding the transference of responsibility to students during modeling activities?

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2. The theoretical framework of the study

2.1. Mathematical modeling

Mathematical modeling represents a process that connects the real and mathematical worlds (Blum & Borromeo Ferri, 2009). Therefore, mathematical modeling is a complex competency that includes overcoming tasks related to the real world (Schukajlow et al., 2015). Modeling competency involves solving real-world problems with mathematical methods (Niss et al., 2007). This competency largely consists of different sub-competencies in the modeling cycle (Maaβ, 2006). However, mathematical modeling competency is more than the sum of these sub-competencies (Schukajlow et al., 2015; Tropper et al., 2015). This competency involves not only cognitive but also metacognitive and affective variables (Schukajlow et al., 2018). Nevertheless, the sub-competencies related to cognition in the modeling cycle are prerequisites for modeling competency (Zöttl et al., 2010). Based on the cognitive perspective, the modeling cycle and sub-competencies used in this study are displayed in Fig. 1.

In Fig. 1, the modeling process is explained by a cyclical process where a situation model is created by understanding the real-life situation, a real model is created by simplifying or structuring the situation model, and a mathematical model is created by mathematizing the real model. The process continues by then working mathematically with mathematical methods, obtaining the real results by translating the mathematical results into the real world, and then validating the real results (Blum & Leiβ, 2007b; Kaiser & Stender, 2013; Maaβ, 2006). This cyclical process is repeated with different variables and assumptions to develop a more valid mathematical model (Blum, 2002; Common Core State Standards Initiative, 2010; Niss et al., 2007). Therefore, mathematical models involve not only mathematical representations such as formulas, equations, and graphs but also conceptual systems that accurately reflect the real world (Czocher, 2017). Studies demonstrate that students have consistently had challenges in the mathematical modeling cycle (Blum, 2011; Fakhrunisa & Hasanah, 2020; Frejd & Ärlebäck, 2011; Galbraith & Stillman, 2006; Gatabi & Abdolahpour, 2013; Ji, 2012; Leiss, 2010; Maaβ, 2006; Schaap et al., 2011; Stillman, 2011; Stillman et al., 2010). When the difficulties in the modeling cycle are examined in detail, failures regarding understanding the task (Schaap et al., 2011); searching for missing information (Maaβ, 2005); identifying relevant variables (Schaap et al., 2011); simplifying assumptions (Stillman et al., 2013); constructing a mathematical model (Galbraith & Stillman, 2006); using proper formulas, solution strategies, and algorithms (Maaβ, 2005); and interpreting mathematical results and identifying the influence of constraints or real-world aspects on the mathematical result (Galbraith & Stillman, 2006) are observed. In addition to difficulties encountered during the modeling stages, the modeling process involves metacognitive, affective, social, or organizational problems (Wess et al., 2021). There is even a potential barrier to understanding the nature of the modeling tasks (Jankvist & Niss, 2018). Researchers have hence revealed that the mathematical modeling process is quite complicated for students and that they need teacher support to become independent modelers (Stender, 2018; Stender & Kaiser, 2015). Teachers' competencies to support students in mathematical modeling activities are thus significant (Wess et al., 2021).

2.2. Teacher competencies in mathematical modeling

Teachers play a significant role in fostering the modeling competencies of students (Greefrath & Vorhölter, 2016). They need different skills, knowledge, and ideas for teaching, as well as appropriate attitudes and beliefs (Wess et al., 2021). There are typical competencies for teachers in managing mathematical modeling processes (Borromeo Ferri, 2018; Borromeo Ferri & Blum, 2010). Wess et al. (2021) defined modeling-specific pedagogical content knowledge, which involves knowledge about the modeling process, modeling tasks, aims or perspectives, and interventions. The interventions include the characteristics and effects of proper support. Particularly, minimal (Alfke, 2017), adaptive (Makar et al., 2015), strategic (Stender & Kaiser, 2015) and diagnostic support (Stender et al., 2017) are considered to have a high potential for fostering students' competency in the modeling process. However, for complex modeling problems, the answer to whether support is appropriate is not easy (Greefrath & Vorhölter, 2016). Teachers' role in the classroom can affect students' modeling performance. This idea is the focus of current research.

Burkhardt (2006) and Doerr (2007) identified a changing role for teachers in supporting the development of modeling competency, unlike traditional treatment. Tabak and Baumgartner (2004) identified three participation structures to reflect the different teacher roles in the classroom: the monitor participant structure, the mentor participant structure, and the partner participant structure. In the first structure, the teachers are in a position of authority over the process. In the second structure, the teachers are also in a position of authority, but they offer more opportunities to the student regarding participation. In the third structure, the teachers share authority with their students (Forman et al., 2017). The teachers present themselves as a peer in contrast to the mentor participant structure (Tabak & Baumgartner, 2004). Studies show that the participation structure of the teacher as a partner is more effective in fostering students' skills (Sun et al., 2022). However, there are open questions about how teachers using different participation structures support students, whether support gradually decreases, and whether students can complete a given modeling task independently. Answering these questions can provide crucial details about how students effectively gain modeling competencies (Durandt & Lautenbach, 2020; Schukajlow et al., 2015; Stender et al., 2017). These questions are

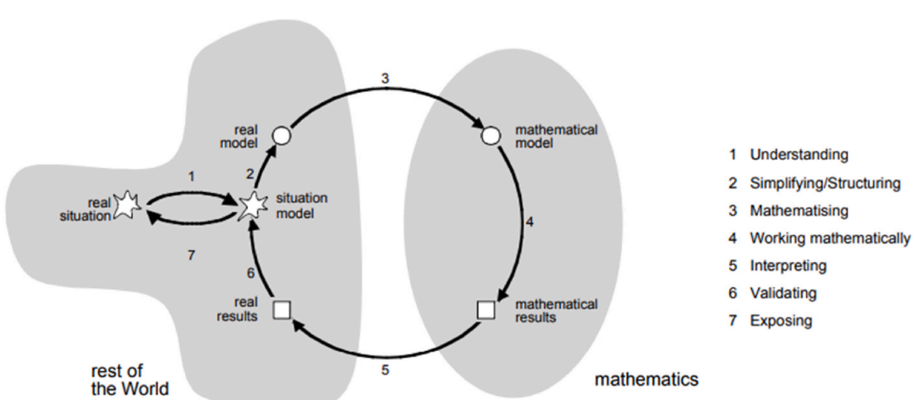

Fig. 1. Mathematical modeling process according to Blum and Leiβ (2007a, p. 10).

examined under the theoretical framework of scaffolding, which has become increasingly popular in education (Bakker et al., 2015).

2.3. Scaffolding

This study discusses the norms of scaffolding in mathematical modeling activities. Scaffolding is based on sociocultural theory, which includes the social foundations of learning. The concept is linked to the zone of proximal development (ZPD), which defined the difference between assisted and unassisted student performance by Vygotsky (1978). One of the examples of ZPD in practice is the concept of scaffolding (Puntambekar, 2022; Wood et al., 1976). This concept was first defined by Wood et al. (1976, p. 90) as "the process that enables a child or novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts". Based on this definition, scaffolding is the temporary support that a knowledgeable person supplies to help students solve problems that they cannot solve independently (Makar et al., 2015; Van de Pol et al., 2011). Scaffolding has been used as an effective method in education (Azevedo & Hadwin, 2005; Davis & Miyake, 2004; Hadwin et al., 2005; Mercer et al., 2004; Pawan, 2008; Shen, 2010; Van de Pol et al., 2010, 2015) and mathematics education in particular (e.g., Fisher et al., 2013; Schukajlow et al., 2012; Smit & Eerde, 2013; Smit et al., 2016; Sunaryo & Fatimah, 2020) for many years. The conceptual model of scaffolding is given in Fig. 2.

As seen in Fig. 2, scaffolding has three characteristics (Van de Pol et al., 2010). The first is the concept of contingency, which is defined as temporary support. Diagnostic strategies and scaffolding strategies are support tools related to contingency. With these strategies, students can reach their potential level of competence, and teachers may reduce their support over time. The second important characteristic is called fading. This term describes the gradual withdrawal of support. Finally, the transfer of responsibility is the third characteristic in which the student begins to carry out the task independently as responsibility is transferred from the teacher to the student. (Van de Pol et al., 2014).

There are various scaffolding tools in the literature, which are called scaffolds. Some examples of scaffolds are dialogs between teachers and students, solution plans, digital learning environments, or simply basic materials (e.g., Abdu et al., 2015; Bywater et al., 2019; Dove & Hollenbrands, 2014; Edson, 2017; Kolikant & Broza, 2011; Pfister et al., 2015; Schukajlow et al., 2015; Sun et al., 2021). Azevedo et al. (2004) classified these tools into either fixed or adaptive scaffolds. A fixed scaffold is static and does not change according to the student's needs. A solution plan can be an example of a fixed scaffold (e.g., Schukajlow et al., 2015). An adaptive scaffold, however, is dynamic, and the support is appropriated according to the student's needs. For instance, teaching through dialog and inquiry-based teaching methods are adaptive

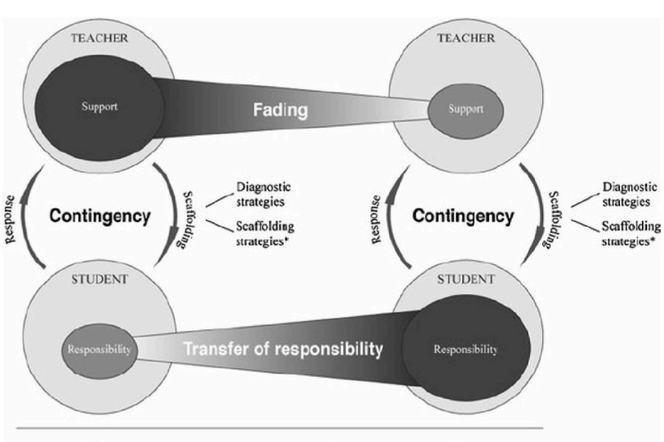

Fig. 2. The conceptual model of scaffolding (Van de Pol et al., 2010, p. 274; p. 274).

scaffolds (e.g., Makar et al., 2015). In adaptive scaffolds, there can be any level of support according to the needs of the individual, such as giving feedback, offering clues, giving explanations, suggesting a model, asking questions, or using diagnostic strategies (Van de Pol et al., 2010). Stender and Kaiser (2015) analyzed the different levels of support in their study. The first and lowest level of support is motivating support, the second is feedback support, and the third is strategic support. Strategic support in mathematical modeling is usually provided by outlining the modeling steps, such as identifying variables, making assumptions, and creating a mathematical model. The fourth is content-related strategic support, which means support by providing more information about the content of the task. The fifth is content-related support, which means that teachers give direct instructions or do the task themselves. These forms of support range from the lowest to the highest level. The transition from higher levels to lower levels or even the complete disappearance of support are indicators of scaffolding (Van de Pol et al., 2015).

2.4. Scaffolding studies in mathematical modeling

Scaffolding is a new concept in teaching mathematical modeling (Durandt & Lautenbach, 2020; Schukajlow et al., 2015; Stender et al., 2017; Stender & Kaiser, 2015). Including heuristic worked examples (Hänze & Leiss, 2022; Zöttl et al., 2010), problem-solving methods (Greefrath, 2015; Stender & Kaiser, 2015), an independence-oriented teaching style (Durandt et al., 2022), dialogic teaching (Stender, 2018; Stender et al., 2017; Tropper et al., 2015), increasing learning aids (Alfke, 2017), digital learning environments (Daher & Shahbari, 2015), or a solution plan (Beckschulte, 2020; Durandt & Lautenbach, 2020; Schukajlow et al., 2015) in the modeling process is an opportunity to support students. Some studies on mathematical modeling have examined the fixed scaffold approach in teaching modeling. In these studies (e.g., Beckschulte, 2020; Durandt & Lautenbach, 2020; Schukajlow et al., 2015), a solution plan describing the modeling cycle was provided to the students, who were then asked to create a model using the solution plan. Students' modeling competencies were thus somewhat improved. The students were also provided with heuristic worked examples (Hänze & Leiss, 2022; Zöttl et al., 2010) as a fixed scaffold. The results confirmed that although the students' modeling competencies could be improved through support, the long-term effects of support were minimal. Although these studies determined that the fixed scaffold is effective in fostering modeling competencies, it was found insufficient for students to complete the modeling process. Nevertheless, some studies on mathematical modeling have examined the adaptive scaffold. In these studies, dialogic teaching (Stender, 2018; Stender et al., 2017; Stender & Kaiser, 2015; Tropper et al., 2015) or increasing learning aids (Alfke, 2017) were used as a scaffold; adaptive scaffold was found highly effective in fostering modeling competencies. Stender et al. (2017) realized that the adaptive support of teachers educated in scaffolding had long-term success in mathematical modeling. In addition, they found that this effect varies according to teachers' experience. Stender and Kaiser's (2015) study indicated that teachers with scaffolding knowledge mostly use strategic and content-related strategic support and that these forms of support foster modeling competencies. Tropper et al. (2015) found that teachers without scaffolding knowledge generally provide students with direct content support in modeling activities. This support is the highest form of support. These studies classified teachers' forms of support according to their scaffolding training and experience. Conversely, this study aimed to reveal the forms of support in modeling according to participation structures. Further research is required to conceptualize the structures of the different scaffolding types in mathematical modeling (Bakker et al., 2015). Classifying the forms of support according to different participant structures can also be important in conceptualizing scaffolding. Furthermore, some studies indicate that fading and the transfer of responsibility are generally lacking in studies on scaffolding (Bakker et al., 2015; Belland, 2011; Lin et al., 2012). This study describes the scaffolding process in mathematical modeling by including the three characteristics of scaffolding (i.e., contingency, fading, and transfer of responsibility).

3. Method

3.1. Research model

This study uses phenomenology design as a qualitative research method. The foundations of this research design are based on Husserl (1970), who proposes phenomenology as a philosophical approach to defining and classifying personal experiences. This method focuses on people's daily experiences and how these experiences are then interpreted (Mayring, 1996). Naturally, the interpretation of these experiences depends on the person's beliefs, academic background, and different social characteristics. This research aims to show how teachers who have different participant structures, experiences, and academic backgrounds support their students during mathematical modeling activities in their classroom. For this reason, in accordance with the purpose of the research, teachers are expected to have experience with mathematical modeling activities in their classrooms and to have communicated with students one-to-one in this process.

The phenomenological design aims to find common points, expressed as the core of their experiences, about how different people perceive or interpret similar experiences (Fraenkel & Wallen, 2006). In this context, this study aimed to reach the commonalities of the scaffolding experiences of teachers with different participant structures and determine which scaffolding methods differ among teachers. Moreover, in-depth interviews offer researchers flexibility for revealing the phenomenon and experience in phenomenological studies (Richards & Morse, 2007). For example, teachers often state that they do not support students. They may have this perception because they interfere with students less in modeling activities than traditional teaching. However, conducting in-depth interviews in phenomenological studies facilitates defining teachers' main roles in the classroom and demonstrating the support they provide.

3.2. Research group

The research group consists of four secondary school mathematics teachers who have applied mathematical modeling activities in their classrooms. In this context, a purposeful sampling method has been used to select the research group (Patton, 2002). For this study, teachers were contacted via e-mail and asked the following questions:

1) "What is mathematical modeling?"
2) "Have your students encountered mathematical modeling before?"
3) "Have you applied modeling tasks in your class?"
4) "If you have applied modeling tasks, how many times?"
5) "Can you write about these tasks?"

Seventeen teachers responded to this e-mail. Eleven teachers defined mathematical modeling as modeling mathematics (see Cirillo et al., 2016). Mathematical modeling was included in the secondary school curriculum in Turkey in 2017; nonetheless, some confusion about modeling in the curriculum remains. These 11 teachers were therefore excluded from the study. Although six teachers correctly defined mathematical modeling, one teacher had not applied a modeling task, while the other had but only once. The remaining four teachers described mathematical modeling accurately and had used more than one application in secondary school classes. Furthermore, their modeling tasks adhered to Wess et al.'s (2021) criteria. The four secondary school mathematics teachers are affiliated with the National Education Ministry in various provinces of Turkey and have met the criteria given below to participate in this study. The criteria are as follows:

- The participants should be secondary school mathematics teachers who have applied mathematical modeling activities in their classrooms. (This is significant because experience is the main criterion in phenomenology studies.)
- The second important criterion is that teachers distinguish between the concepts of mathematical modeling and the modeling of mathematics. In this respect, during the interviews with the teachers, they were first asked to define modeling. The teachers who characterized modeling as a special method of solving daily life problems were prioritized.
- It was required that the teachers had applied the modeling activity at least twice with the same group during the process. This enables students to comment on the development of their modeling competencies.
- The last criterion is voluntary participation in the study. Due to the long duration of the interview process, voluntary participation in the study is thought to affect the question-and-answer process positively. The demographic information of the teachers is given in Table 1.

As seen in Table 1, an equal number of male and female teachers participated in the study. The years of teaching experience ranged from 1 to 15 years. All the teachers worked in different villages with low socioeconomic status. Each teacher had completed or was about to complete their master's thesis and has specifically worked on mathematical modeling. Except for one teacher (T1), other teachers did not receive any formal training in modeling but learned about this area from their thesis research, other articles, and their advisors. The teacher who was formally trained had taken a mathematical modeling course at the undergraduate level for one semester. The teachers applied the modeling activities at different secondary school grade levels. According to the teachers, the students who participated in the study were at a low to medium academic level. One teacher said that she used mathematical modeling activities twice, another teacher three times, another four times, and the other teacher 32 times. When the teachers were asked where they obtained their modeling tasks, they stated that they usually prepared them themselves and thus applied original modeling tasks. The tasks were also examined for reference to reality, relevance, authenticity, openness, and promoting modeling competencies, which are simplifying, mathematizing, working mathematically, interpreting and validating (Wess et al., 2021). They were also confirmed as modeling tasks by both the researcher and an expert in the mathematical modeling field. In addition, the fact that the teachers had used these tasks in the research for their master's theses is an indicator that the tasks were prepared according to modeling task criteria. Altogether, 41 tasks were applied. One teacher also used modeling videos from a website. For example, T1 used the following task: "*The active volcano exploded, and the lava started to move towards the city. When will the lava reach the city, if that's possible?" (The modeling task is presented in video form)*. Other teachers created modeling tasks that considered the students' real lives. For instance, T4 used the following task: "*You spend some time waiting in line for lunch at school. How can you shorten the waiting time of students to use your lunchtime more effectively and efficiently? Can you find a solution mathematically?" (The teacher said that students have this problem in real life)*.

The teachers did not receive training on scaffolding but supported the mathematical modeling process with purely intrinsic motivation. They said that they avoided directing students in mathematical modeling, while all teachers used traditional methods in teaching mathematics. In the interviews with each of the teachers, they all stated that mathematical modeling applications contributed to the development of their student's mathematics education and that they wanted to continue applying mathematical modeling in the future. Therefore, they all have positive views of modeling.

Table 1
Teachers' demographic information.

Teacher	Gender	Years of experience	Where they work	Education level	Knowledge about Modeling	Field of study for their thesis	The grade level of the modeling activity	Number of times modeling activity is done	Last time it was done	Modeling task	Scaffolding related training
T1	Female	1 year	Village	Masters	Have knowledge	Modeling	7th grade	2 times	April 2021	Existing	Not received
T2	Male	6 years	Village	Masters	Have knowledge	Modeling	7th and 8th grades	32 times	April 2021	Original	Not received
T3	Female	15 years	Village	Masters	Have knowledge	Modeling	6th grade	3 times	November 2019	Original	Not received
T4	Male	5 years	Village	Masters	Have knowledge	Modeling	7th grade	4 times	March 2021	Original	Not received

3.3. Data collection tools and data collection

In this study, it was used a semi-structured interview technique which is a qualitative data collection method. This interview technique provides the researcher with the flexibility to conduct more in-depth research (Creswell, 2005). An interview form with 14 questions was prepared by the researcher (see Appendix A). While the interview form was being prepared, the literature on mathematical modeling was initially reviewed. Since it is stated in the literature that the term 'mathematical modeling' is often confused with the concept of the modeling of mathematics (Cirillo et al., 2016), the teachers were first asked how they defined the term. At the same time, the literature on scaffolding was carefully surveyed. The literature shows that there are different forms of support (Anghileri, 2006; Smit & Eerde, 2013; Stender & Kaiser, 2015). As a result, interview questions were prepared to reveal whether the teacher provided support. Great care was taken to ensure that the questions did not contain any implicit direction. For example, instead of asking "*Do you provide motivating* support *to students during the activity?*", the questions "*Can you explain a day when you implemented the modeling activity*?" or "*What do you do in class during the activity?*" were used (see Appendix-A). Finally, in accordance with phenomenology design, demographic information is also included to discuss the scaffolding methods of teachers with different participant structures, experiences and backgrounds.

A draft form of the interview questions was sent to experts who previously worked in the field of phenomenology. Questions that were thought to contain implicit direction were corrected in accordance with the expert's opinion. A pilot test of the draft interview was conducted with a teacher. The interview took approximately 50 min. Subsequently, the order of some questions was changed to be more suitable for the flow of the conversation. It was intended that the data would be collected during one-on-one interviews with each teacher. However, since it was not possible to meet face-to-face due to the pandemic, interviews were conducted via an online platform. The interviews lasted between 45 and 75 min. All interviews were audio recorded.

3.4. Data analysis

The data were analyzed based on deductive and inductive methods, which are techniques used in qualitative content analysis (Mayring, 2014). Three participation structures that Tabak and Baumgartner (2004) identified are classified in terms of teacher-student interaction. The current study aims to explain three participation structures with support areas and levels. For the first subproblem, each teacher's participation structures were examined using the deductive method since the low number of teachers participating in the study did not facilitate identifying new participation structures. However, all teacher types covered existing cases. The other subproblems were examined using the inductive method, which was preferred to reveal the patterns in the teachers' scaffolding experiences. Based on the phenomenological design, this study aimed to identify similar and different points in teachers' experiences with the inductive method. This method attempts to match all the data with the various codes and themes. After all audio recordings were transcribed, open coding was completed by the researcher. Then, these codes were combined to create themes and axes.

For the first subproblem, the teachers were asked the following questions to determine their participation structures. For example, "*Starting from when you entered the classroom, what did you do? What do you do at the beginning? What do you do during the activity? What do you do at the end of the activity? How do you communicate with students during modeling activities?*" (See Appendix-A for more). The data were classified under the three participant structures classified by Tabak and Baumgartner (2004). The participant structures are presented in Table 2.

The data relating to the first subproblem of the study have been classified according to the participant structures in Table 2. This subproblem reveal how the teachers positioned and defined themselves during mathematical modeling activities.

For the second subproblem of the research, the teachers were asked the following questions to determine the students' support requests and teachers' reactions. For example, "*Do they want* support *from you? What kind of questions do they ask you? How do you manage the question-and-answer process? Can you give an example?*" (See Appendix-A for more). Then, the levels and areas of support provided by the teachers were classified to determine the "contingency" characteristic of the scaffolding in Fig. 3.

As presented in Fig. 3, the teacher support areas during modeling activities are grouped under three categories: *teacher* support *in organizing the learning environment, teacher* support *regarding the nature of*

Table 2
Participant structures according to Tabak and Baumgartner (2004).

Monitor	The role of the teacher is to establish tasks and procedures and ensure that they are carried out; the role of students is to carry out actions and provide information about their progress. The teacher can answer and teach the students directly. Therefore, the teacher is in a position of authority. The teacher is the initiator of the process and the student is the one who responds. The teacher is physically located outside the group circle.
Mentor	The teacher tries to help the students without directly teaching or giving explanations. They use questions such as "why" and "for what" to encourage students to make systematic comparisons throughout the process. The student participation rate is high. Again, the teacher is in a position to initiate the process and the student is in a position to respond. Therefore, the teacher is partially in a position of authority. They are physically located outside the group circle.
Partner	The teachers present themself as a peer as opposed to the other participant structures in which they present themself in a position of authority. The teacher joins the group for a few minutes and acts as a real member of the group. Teachers and students share roles in the classroom. The teachers' participation in the process is as a partner with the students. Teachers are physically located within the group circle.

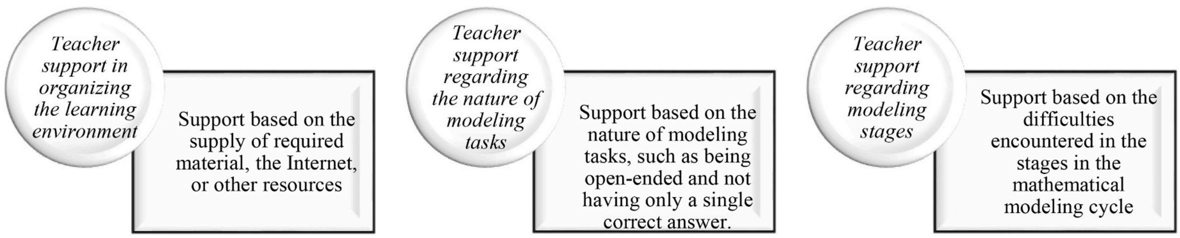

Fig. 3. Teacher support areas during modeling activities.

mathematical modeling tasks, and teacher support *regarding modeling stages*. These support areas were extracted from the teacher statements with the inductive method. Each of the support areas was examined according to the levels of support described by Stender and Kaiser (2015). Teacher statements were coded according to levels of support. They were expanded when the teachers' answers did not fit the existing codes. It added new levels of support to the classification. The new codes were associated with the literature (Leiss & Wiegand, 2005; Smit & Eerde, 2013; Stender et al., 2017). These support levels and some additional ones are given in Table 3.

The forms of support given in Table 3 follow a hierarchical order from top to bottom. Motivating support is the lowest level of support, and teaching support is the highest level of support. Three new codes that emerged with the inductive method were added to the theoretical framework of the analysis.

The third subproblem aims to show the fading characteristic of the scaffolding. Teachers were asked the following questions to determine the students' support requests and teachers' reactions. For example, "*How do they behave after* support*? What kind of change do you observe? Can you give an example?* (See Appendix-A for more). One of the indicators of fading is the decrease or complete disappearance of support over time (Smit & Eerde, 2013). Expressions such as "*decrease, no longer* supporting" used by teachers were evaluated as fading. For the fourth subproblem of the research, the teachers were asked the following questions to determine the students' modeling competencies. For example, "*What do you think about your students' mathematical modeling skills? Why do you think this is so?* (See Appendix-A for more). When the teachers expressed a level of *positive change in the students' modeling competencies*, this is identified as the transfer of responsibility.

Studies on the reliability and validity of the research are conducted in accordance with the nature of qualitative research. Toma (2006) proposes that member control is an important way to assess reliability. Two types of member control have been conducted. First, the quotations and the codes are confirmed by an outside expert. Cohen's Kappa coefficient, which assesses interrater reliability, was found to be 0.78. In this case, interrater reliability is satisfactory (Cohen, 1960). Second, the findings and conclusions are discussed with the participating teachers. Direct quotations are included in the findings sections to enhance the validity of the research.

4. Results

In this study, the data are analyzed under four subproblems. These are the participant structure of the teachers, teachers' forms of support, the fading of teacher support over time, and the transference of responsibility to students.

4.1. Teachers' Participant Structures

The first subproblem of the study discusses how the teachers positioned themselves during mathematical modeling activities. The participant structures in this study are shown in Fig. 4.

According to Fig. 4, the teachers have different participant structures. Based on the teachers' statements, T2 acted as a monitor during the modeling activities, while T1 and T3 acted as mentors and T4 shared authority with his students as a partner. T2 noted that he preferred to distance himself from the students during the process in order not to interfere with the students and not put pressure on the students. He said, "*As soon as I entered the class, I gave the students the worksheet, the students read it, I did not intervene in any way, I observed them sitting quietly from a distance at the teacher's desk throughout the process in order not to interfere with them*". T2 positioned himself only as an observer and provided support at the end of the process. He explained, "*After the activity was over, the students were talking about what work they had done. We completed their solution together.* T1 and T3 positioned themselves as interrogators during the modeling activities. Although they are in a position of authority, they reported that they encouraged students to state their ideas. T1 said, "*We talked about why they think this way, how they move the process forward. I asked questions about the solution and expected answers from them.*" T3 noted that "*If something goes wrong, I warned them by saying 'consider it from a different perspective and try different things*". T4 positioned himself as a partner participant in the modeling activities and made the students feel that he was not knowledgeable. T4 said to his students, "*You want to get information from me, but I only have as much*

Table 3
Classification of levels of support.

Forms of Support	Explanation	Example
No support*	–	–
Motivating support	Students are motivated only in a general way.	I'm sure you can do it. Very good, great.
Diagnostic support*	Students are asked diagnostic-based questions to find out what they are doing and why they are doing it.	What did you find? Why did you choose this variable?
Feedback support	Successful intermediates are provided as positive feedback.	What you have done is correct. Continue like this.
Strategic support	Hints on the procedure are provided. These hints are not directly related to the content (problem or task) and are strategic.	How about formulating it? Can you set up an equation?
Content-related strategic support	Procedural hints are provided, and there is content provided with the information.	Calculate number four first, and then do the same calculation using different variables.
Content support	It is content-related support. It depends entirely on the content of the task and includes the essence of the work.	The calculations needed are explicitly shown by the teacher.
Teaching support*	Teaching about the relevant topic, solving exercises, translating the content	Explaining the topic of "patterns", Solving exercises to find the general term

* Newly added support forms.

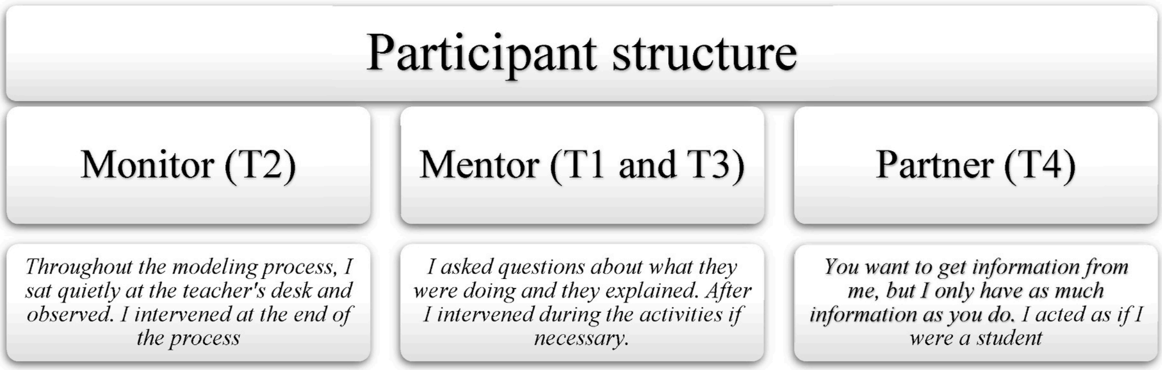

Fig. 4. Participant structure of teachers during the mathematical modeling process.

information as you do". He said, "*I tried to act as if I were a student during the activity. In other words, I pulled up a chair to each group as if I were a student … I would say - so, what are we doing now? -*" Thus, it is seen that T4 did not position himself as the authority in the classroom.

4.2. *Areas and Levels of Support from Teachers*

The second subproblem of the research classifies the areas and levels of support that are given by the teachers.

4.2.1. *Teacher support in organizing the learning environment*

All teachers expressed that the students requested support in obtaining materials, resources from the internet, or other resources. All the teachers met these requests with all that was available. T1 and T3 stated that they allowed the students to benefit from books and the internet. T2 and T3 let them leave the classroom to make measurements and observations. For example, T2 expressed this idea in the following way.

T2: *They wanted to make mathematical measurements; they went out to the schoolyard. Again, they wanted to investigate mathematical concepts and formulas; they went to the computer class and the library.*

4.2.2. *Teacher support regarding the nature of mathematical modeling tasks*

The teachers stated that the students requested support regarding the nature of the modeling tasks. The forms of support given by the teachers regarding the nature of mathematical modeling tasks are presented in Fig. 5.

In Fig. 5, the first form of support is about the lack of numbers in some modeling tasks. For example, T1's students watched a video showing the eruption and spread of lava from a volcanic mountain. In the modeling task used by T1, the lack of numbers caused the students to ask the teacher for support. T1 said that "*The students were very surprised about this: There are no numbers, what are we going to do here? So, I directed them … I stated that you can predict this in your imagination.*" The teacher stated that they could move forward by estimating the numbers themselves; thus, she provided strategic support to help the students understand the nature of the modeling tasks. The relevant quotation is given below.

T1: "*I stated that this is a typical math problem. There are not only variables in the problem. You can do something by giving the variables yourself.*

In the second form of support request given in Fig. 5, the teachers agreed that the students wanted to confirm with their teacher whether their assumptions, variables, predictions, solutions, or mathematical expressions were correct. T2 and T3 stated that they avoided providing support. For instance, T2 expressed that "*The students asked about each thing they had done. They asked continuously, 'Teacher, are we doing it right?'. But I stated that I did not want to intervene in any way*". On the other hand, T4 said that he provided motivating support to help the students understand that "*all solutions are valuable*". In addition, T4 stated that he tried to understand what the students were doing by asking diagnostic questions. The relevant quotation is as follows.

T4: For example, the student comes to me and says, we have created a graph, is it correct? I know that I should not say if it is right or wrong there because I don't want to be directly guiding them in modeling … Maybe there is something else on the graph. So, I asked them questions like how did you create the graph, what did you think?

The third form of support for the nature of mathematical modeling tasks given in Fig. 5 is that students expect the teacher to evaluate which answer is the correct end of the process. According to the teachers' statements, T1 and T2 provided strategic support to the students by saying, "*there is no single result in modeling*". Teachers agreed that the students expect the teacher to explain which answer is correct. They naturally see their teacher as an authority and want their work to be constantly verified by the teacher. The teachers tried to overcome this situation by providing low support to students' requests.

4.2.3. *Teacher support regarding modeling stages*

During the mathematical modeling activities, students requested support from the teacher regarding the mathematical modeling stages, and the forms of support that were provided are presented in Table 4.

According to the teachers' statements, T1 and T3 provided strategic support to the students who had difficulty at the understanding stage, while T2 provided content-related strategic support. T1 suggested that

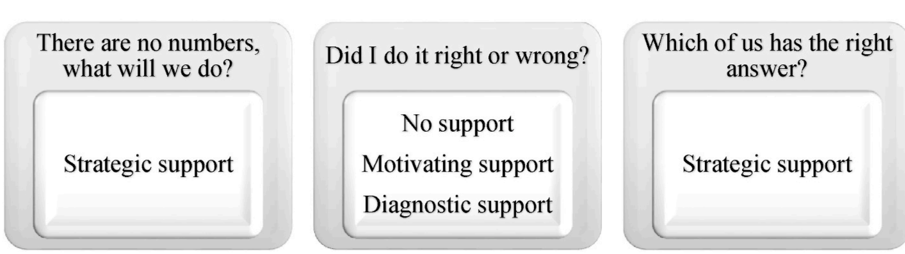

Fig. 5. Teacher support regarding the nature of mathematical modeling tasks.

Table 4
Teacher support regarding modeling stages.

	T1 (Mentor)	T2 (Monitor)	T3 (Mentor)	T4 (Partner)
Understanding	SS*	CRSS*	SS*	–
Simplifying and structuring	NS and CRSS	–	NS and CRSS	MS and CRSS
Mathematizing	–	CS and TS	–	SS and CRSS*
Working mathematically	–	CS and TS	CRSS	–
Interpreting	–	–	–	–
Validating	–	–	–	–

* Teachers stated that students' competencies improved at these stages.
NS: No support; MS: Motivating support; SS: Strategic support; CRSS: Content-related strategic support; CS: Content support; TS: Teaching support.

the students watch videos on the internet about the subject and said, *"The question in the video I showed the students related to a volcano. They wondered how the volcano erupted … I've also said there are other videos like this. You can watch them on the internet*". At the understanding stage, T3 reflected that the students did not know what to do and where to start; therefore, she supported them by saying, "*try to visualize what comes to your mind*". When T2 realized that the students were having problems understanding the situation, he implied that he provided content-related strategic support to them. T2 specified that he supported the modeling task by comparing it to real-life situations that the students have encountered before. T2 said, *"When I gave the problem on the growth of bacteria, I explained it through their experience of Corona (coronavirus)*".

At the stage of simplifying and structuring, based on the teachers' statements, T1 and T3 did not support their student in some cases. T1 said that *when the students searched on the internet, they noted that the lava flowing from the volcano was not in the shape of a full circle*. T1 said that she did not comment on that idea, and the students eventually did not use this idea later. The students again continued assuming that the lava was spreading in a full circle. Similarly, T3 stated that *the students wondered whether an object thrown into the olive oil would change its location according to the shape of the bowl. In this sense, they wanted to determine the shape of the bowl as a variable.* T3 did not comment on this idea, and the students eventually renounced it. *Then, the students tried to show the location by drawing a classic rectangular bowl.* On the other hand, the teachers (T1, T3, T4) were noticed to have mostly used content-related strategic support at the simplifying and structuring stage. For instance, T4 stated that the modelers who were investigating how long it took for students to consume food at the simplifying and structuring stage determined that gender was a variable affecting the task. It was also seen that the teacher provided *content-related strategic* support for determining the grade level as a new variable. T4 expressed this idea in the following way.

*T4: In the cafeteria problem, they considered gender variables as girls versus boys (in terms of the duration of food consumption). Girls eat faster than boys … Do the girls in every grade eat faster than the boys in each grade? I asked a question like this. This time, the students increased their variables and examined the girls in the 5*th*, 6*th*, 7*th*, and 8*th *grades*.

Again, at the simplifying and structuring stage, T4 provided *motivational* support to the students who determined that social relations affected as a variable how long students took to eat food. This support led the students to move to the next level. The relevant quotation is as follows.

T4: One group was stuck. They didn't even realize what was happening. I sat next to them … One member of the group said that when the boys eat together or when the girls eat together, they take longer to eat their meals. I asked 'why'. They said when a girl eats with a boy, they eat faster. I caught this. I told them, great, you are on to something. I looked at the class and said that your friends found something very interesting. Then, this group said let's go, we will eat together as boys with boys and as girls with girls, and I was to keep the time. Then, they made their generalization here. This was huge for me. I mean, the students modeled how social relationships affected the duration of food consumption, and this was done by a group of students that are perceived to be underachievers.

At the mathematizing stage, according to the teacher's statement, T2 who acted as a monitor provided content support and teaching support, which are the highest levels of support. The teacher stated that he provided teaching support to the students who could not create a mathematical model to find the general formula of a pattern. Despite this support, the students could still not find the mathematical model. More specifically, it is ascertained that by giving *content* support afterward, he provided direct support for students in finding the mathematical model. The teacher explained,

T2: For example, in a problem about bacteria, they considered the number of bacteria as a pattern, but they could not form a rule to represent that pattern. I reminded them how we could find the rule for the pattern. First, I showed them how to solve classic math problems like how we find the rule for the pattern. Again, they had difficulty forming the rule for the pattern. I had to intervene. I tried to establish the rule for the pattern myself in their model to a certain extent.

In contrast, T4 implied that he provided strategic support to the students who had difficulties at the mathematizing stage by saying, *"How can you explain this mathematically? There are so many things you have learned. You know algebra, you know equations"*. He stated that after the support provided, the students looked at the problem from a more mathematical perspective and made the transition into the world of mathematics. T4 provided content-related strategic support to the students so that they could generalize the mathematical models. The students created a mathematical model representing how long it takes for a student to consume food. T4 said that "*the students researched the duration of food consumption for a boy and a girl"*. The teacher said that he warned them that data from several people are required to create a model by saying "*You eat in a different manner (duration). He/she also eats in a different manner (duration)*". Then, he said that the students observed more people eating. He said, *"They add up the total duration of time that the girls ate and divide it by the number of girls. So, I saw that they used the arithmetic average*".

At the stage of working mathematically, it is understood from the teacher's statement that T2, who acted as a monitor, provided content support and teaching support to the students at the end of the modeling process. The teacher noticed that there were deficiencies in the students' prior knowledge about percentages. T2 said, "*I solved the basic problems by explaining the subject of percentages (teaching* support*) … For example, the students did not find 20% of 500 and they had difficulties. I said, you write twenty percent as a fraction and you will multiply those two numbers (*content support*) …"* On the other hand, it is noticed that T3 provided content-related strategic support to the students specifically about the ordering of fractional numbers when they were at the stage of working mathematically. The relevant quotation is as follows.

T3: *(ordering fractions) They have to equalize either the denominators or equalize the numerators. They were directly considering this way. Then, they failed to order the fractional numbers correctly. For example, they have 1/33 and 2/67. How were they to order them? They wanted to equalize the denominators no matter what. I said that you can think about ordering the fractions in every sense. You can think of its proximity to the half, its proximity to the whole, its proximity to zero. You can convert it to a decimal. You can convert it to a percentage*.

It is understood that T1 and T3 use similar forms of support at the stages of understanding and simplifying and structuring the situation. Based on the teacher's statement, T2 did not intervene during the modeling activities of the students, but at the end of the activity, he used content support and teaching support with the students, which are high-level support forms. T4 noted that he used motivating support, strategic support, and content-related strategic support, which are low-level forms of support. It is noteworthy that the teachers never mentioned support at the stages of interpretation and validation.

4.3. Teacher Support Fading Over Time

The third subproblem examines the fading characteristic of scaffolding. Based on the teachers' utterances, the support from teachers faded with respect to the nature of the mathematical modeling task. It is noticed by all teachers that there were three support requests in this area. First, the lack of numbers in some modeling tasks caused the students to ask for support. T1 said that she provided strategic support by saying "*You can guess the numbers yourselves*". Then, she stated that the students became accustomed to the nature of the modeling tasks even by the second activity.

Second, the students wanted confirmation from the teacher. They asked, "*Did I do it right? Is this graph correct, teacher? Is this variable we chose, correct?"* T4 mentioned that he responded to the student's request for verification with diagnostic questions, saying, "*When they asked such questions, I replied to their question with another question*". The fact that students did not receive a direct answer as to whether their answer was right or wrong is assumed to have caused less of a desire for teacher verification over time. T4 stated that fading occurred and commented that "*This (request for verification) was more at the beginning of the study, and then it decreased a little bit.*" T3 said that she did not respond to students' requests for verification at all. T3 stated that students' requests for verification decreased over time by saying, "*[They realized] their resistance was hopeless and that the teacher does not help. I have to deal with this completely on my own; I have to struggle with this.*"

Third, it is said that the students wanted to know whose answer was correct at the end of the process. It is understood that T2 and T4 provided strategic support by saying "*there is no single answer to these questions*". At the end of the first activity, T4 said that he accepted all of the solutions given by the students and helped the students understand that the nature of modeling is different from standard mathematics problems. The relevant quotation is as follows.

T4: They presented their solutions to the first problem, and we accepted each solution. When they were working on the second problem, they were more relaxed, knowing that there was not a single correct answer.

All these statements show that the support from the teacher faded over time when the students were working through the modeling tasks. There was no indication from the interviews that the support requests related to organizing the learning environment and understanding the mathematical modeling areas faded.

4.4. Teacher's Observations about The Transference of Responsibility to Students

The fourth subproblem of the research examines the characteristic of scaffolding called the transfer of responsibility. The development of students' modeling competencies is an indication that responsibility has been transferred. In this respect, the modeling competencies of the students according to the teachers are discussed. According to Table 4, three of the teachers who provided support said that the students' understanding skills improved. A sample excerpt from T1 is as follows.

T1: Even the second activity was different. I entered the classroom with a laptop. Everyone was delighted. Yay, we are going to do an activity. When I started the video, they didn't expect me to explain anything about the problem (the modeling task with no numbers). They already understood how the task would proceed.

It was understood from the teacher's utterance that T4 provided strategic support and content-related strategic support to the students at the mathematizing stage. The teacher stated that the students' mathematizing competence had improved. The relevant quotation is as follows.

T4: There was a change in the students. For example, they had difficulties during the mathematization stage in the first (activity). In the second (activity) I saw that the transition to the mathematization stage was faster and that the students were able to think the situation more mathematically … And the results were a little better.

On the other hand, although T2 said that he provided content support and teaching support, which are high-level forms of support to the students at the stage of mathematizing, the teacher did not observe any improvement in the students. The relevant quotation is as follows.

T2: The students cannot develop any mathematical models. They can't form any equations. They can't generalize it. From the beginning to the end of all activities, the students could not develop any mathematical models.

5. Conclusion and discussion

This study examines the scaffolding methods of teachers who used different participation structures during mathematical modeling activities. The teachers agree that their students need support in the same areas. According to the teachers' independent explanations, all the teachers provided support in the same areas, even if their participation structures differed. In this study, the support areas were common points expressed as the core of their experiences (phenomenon). Nevertheless, teachers with different participant structures tended to use different levels of support. Based on the teachers' utterances, the teacher who acted as a monitor provided the highest levels of support to the students at the end of the modeling activity, while the teachers who acted as mentors provided a lower level of support than a monitor. The teacher who acted as a partner offered the lowest levels of support. Three teacher types that Tabak and Baumgartner (2004) identified are classified in terms of teacher-student interaction. However, the explanation of three participation structures with support levels is a new result. The support levels can be correlated with teachers' participation structures.

Based on independent teacher interviews, three support areas were identified in this study. First, all the teachers said that the students asked for support in organizing the learning environment. Anghileri (2006) defined organizational support as a scaffolding strategy at the lowest level. None of the teachers remarked on the fading of this support. Whether the support involved in environmental needs can be considered scaffolding thus remains unclear. Indeed, some studies (Puntambekar & Hübscher, 2005; Smit & Eerde, 2013) have addressed whether this support area is scaffolding because no fading occurs. Nonetheless, even if it is not scaffolding, it is an effective support tool (Anghileri, 2006).

Second, based on the teacher statements, students who encounter modeling tasks for the first time might have difficulties with the nature of mathematical modeling tasks, as Jankvist & Niss, 2018 noted in their study. Three different challenges arose, according to teachers. The first challenge was, *"There are no numbers! What are we going to do now*?" The second one was, "*Are we doing it right*?" The third one was, "*Which of us has the right answer*?" Responding to the first challenge, the teachers stated that they provided strategic support to the students, saying, "*You can make assumptions, you can guess, you can visualize it*." For the second challenge, the teachers said that they did not assist the students when asked for confirmation but asked diagnostic questions, such as *"What did you do? How did you do it? Why did you choose this method?"* Finally, for the third challenge, the teachers implied that they provided strategic support, such as "*There is no single answer in modeling tasks"*. In each case, the teachers stated that the frequency of the students' requests for support faded over time. Support in this area may have faded over time through strategic or diagnostic support. However, working on modeling tasks for a long time may have contributed to the fading of the support. Stillman et al. (2015) stated that participation in modeling activities positively affects difficulties with the nature of mathematical modeling tasks. In the current study, the teacher who practiced for a long time and the teacher who practiced for a short time stated that the students' support requests decreased.

Third, the forms of support related to the mathematical modeling stages are classified in this study. Particularly, the support levels differ in this support area. According to the teachers' answers, the teachers who acted as mentors provided strategic support, while the teacher who acted as a monitor provided content-related strategic support at the stage of understanding. All of them claimed that the students' understanding competence improved. The teachers who acted as mentors said that they did not support some students' new and original assumptions at the simplifying and structuring stage. They claimed that these students renounced these assumptions later, which may mean that these teachers do not provide support tailored to the students' needs (Van de Pol et al., 2022). Broza and Kolikant (2015) indicated that students' learning processes include many regressions alongside their progressions. In these cases, teachers' lack of support can lead students to renounce their different ideas. However, the teacher who acted as a partner led the students to progress by offering motivational support to their original assumptions. In addition, teachers who acted as both mentors and partners tended to provide content-related strategic support to help students progress at this stage. Leiss (2010) and Tropper et al. (2015) determined that teachers use the most content-related strategic support during the simplifying and structuring stage. Furthermore, based on the teachers' utterances, the most support was provided during the mathematization stage. The teachers stated that the students did not generalize the mathematical models or explain them mathematically. The teacher who acted as a partner tended to provide a lower level of support to the students at the mathematizing stage and he said that the students' mathematizing competence improved. Although the teacher who acted as a monitor tended to provide a higher level of support to the students at the stages involving mathematizing and working mathematically at the end of the modeling process, based on the teacher's observations, the students' competencies remained at the same level throughout the modeling activities. Stender and Kaiser (2015) found that more experienced teachers in modeling and scaffolding provided more strategic support to their students than non-experienced teachers. Nevertheless, Tropper et al. (2015) and Stender et al. (2017) specified that teachers without knowledge of scaffolding tend to provide a higher level of support, while teachers with knowledge of scaffolding provide a lower level of support. In this study, while all the teachers were knowledgeable about modeling but not scaffolding, the teacher who acted as a monitor provided a higher level of support to the students. However, the teacher with a similar background who acted as a partner provided a lower level of support. This result emphasizes the relationship between the participant structure and the support level. Finally, the teachers did not provide support at the interpretation and validating stages. Nonetheless, the literature (Biccard & Wessels, 2011; Blum & Leiβ, 2007b; Gatabi & Abdolahpour, 2013; Zawojewski, 2013) verifies that students have problems at the validating stage. The lack of support at the interpretation and validating stages can be associated with the students' inability to perform these stages.

In summary, teachers' participation structures can relate to their support levels but not support areas. Based on the teachers' utterances, the teacher who acted as a monitor provided a higher level of support, which was ineffective in developing students' modeling competencies, whereas the teacher who acted as a partner provided a lower level of support, which was more effective in developing students' modeling competencies. Teachers who acted as partners or mentors tended to provide strategic support and content-related strategic support. Providing strategic and content-related strategic support seems to be an effective scaffolding method for fostering students' modeling competencies. Similar results on the importance of these forms of support are also noted in Stender's (2018), Stender and Kaiser's (2015), Durandt and Lautenbach's (2020), and Schukajlow et al.'s (2015) studies.

6. Practical results and future studies

This paper argues that teachers' participation structures relate to their scaffolding methods. Similarities and differences between the scaffolding methods of teachers with different participation structures occur. Specifically, similar points concerning support areas and different points concerning support levels have been found. In this context, the study's results hope to contribute to the field in two ways: First, the different support levels may be related to the teachers' participant structures, and the support levels may therefore be included in the characteristics of participation structures that Tabak and Baumgartner (2004) classified. Second, the similar support areas may be an essential scaffolding component in modeling activities. This result provides important details for teachers with different participation structures to support their students in developing modeling competencies. Based on the common opinion of three types of teachers, providing support in organizing the learning environment, guidance on the nature of the modeling tasks, and help with the mathematical modeling stage can be valuable components in supporting the modeling process. The teachers agreed that their students face difficulties in similar areas and need support. However, teachers' reactions to support requests differ. Strategic support and content-related strategic support appear to be effective scaffolding methods for fostering students' modeling competencies. A partner (or at least a mentor) participant structure in the modeling process is an important teacher role in the preference for these scaffolding methods. In this context, teachers' participant structures can be improved to guide students successfully during modeling activities. Improving participant structures could help prevent teachers who act as monitors from using higher-level forms of support that are not as effective. Training on support levels can be an effective component in improving teachers' participation structures. In addition, knowing the support areas that exist as a common phenomenon beforehand is crucial for scaffolding in mathematical modeling.

This study involves analyzing teachers' statements because such a study allows more teachers who used different participation structures to be interviewed and facilitates an analysis of the teachers' diverse perceptions. However, this study's results are limited to the realities of a phenomenological study. Future studies can compare these results with those obtained through direct observation. This study's other limitation was that only teachers who used three participation structures were interviewed. Future studies could aim to reveal the different roles of more teachers and the scaffolding methods connected to these roles. Moreover, this study only reflected teachers' perceptions. Scaffolding is a method that requires examining the teacher's behavior and the student's skill (Smit & Eerde, 2013). In this study, a limited number of teachers' statements on students' competencies were received. Students' modeling skills were therefore limited to teacher perceptions. Measuring students' competencies in future studies might offer more information. These research results could inspire more research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

Appendix-A. Interview form

1. What is your education level? Which faculty/department are you a graduate of? Are you pursuing graduate studies? What level are you at? What kind of courses did you take? If you are studying for

a graduate degree, what area are you studying? How many years have you been in the teaching profession?
2. Where do you teach (province, district, village)? What do you think about your students' overall math success? Why do you think so? What do you think about the socio-economic situation of students? Why?
3. Have you received any training in mathematical modeling? (For example, in a course or in-service training). Can you tell us about its content? How has it influenced your perspective on mathematical modeling? How do you explain mathematical modeling?
4. Do you practice mathematical modeling activities in your classes? How often do you practice it? How many times have you practiced it? When was the last time you practiced it? How long does a modeling activity last on average?
5. How do you prepare modeling problems? How do you create/write the problem? How do you choose the problem? Did you have any other criteria? Are there any books, websites, etc. Resources you use? If so, can you tell us these sources?
6. Can you tell us about the process of modeling activities in your classroom? Starting from when you entered the classroom, what did you do? What do you do at the beginning? (dividing the groups, distributing the modeling problem, talking about it, warm-up stages) What do you do during the activity? (Walking around the class, watching them, communicating with groups, providing feedback, etc.) What do you do at the end of the activity? (asking them to share their models, providing feedback, motivating them, enabling them to discuss, making them compare the models, etc.)
7. How do you communicate with students during modeling activities? Do they want support from you? What kind of questions do they ask you? How do you manage the question-and-answer process? Can you give an example? Who initiates the question-and-answer process? Do you ever support them or provide explanations without them asking for it?
8. When do they request support? How do they communicate the request? (In or out of the classroom, by communicating the request publicly and immediately, asking after the lesson, etc.)? Do they communicate after discussing it with the group, or as soon as they think of it?
9. How do they behave after support? What kind of change do you observe? Can you give an example? Was there a change in their modeling competencies?
10. Which situations do they want support for? Which situations do you not receive requests for support or questions?
11. How do you evaluate the student's answers in modeling activities? What do you pay attention to when evaluating them?
12. What do you think about your students' mathematical modeling skills? Why do you think this is so? How do you react to students who are successful? How do you react to students who are unsuccessful? Is there any difference?
13. Do you make use of any method, technique, or technology while doing a mathematical modeling activity? If so, can you tell us about them? Is there an application or method that you think is particularly appropriate to use?
14. What do you think about mathematical modeling practices in general? How do you evaluate them, especially in terms of education? Do you intend to continue to use these activities?

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